Mathematics: analysis and approaches
Higher level
Paper 2

Name

worked solutions

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

16 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (56 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

On the first day of 2024, Noah deposited h dollars in a bank account that earns a nominal annual interest rate of 4.8% compounded **monthly**. Interest is added to the account on the first day of each month. Interest is added for the first time on 1st February 2024.

The amount of money in Noah's account on **the first day of each year** follows a geometric sequence with the common ratio r.

(a) Find the value of r, giving your answer to four significant figures. [3]

(b) If Noah makes no further deposits into or withdrawals from the account, find the **year and month** when his account has at least $2h$ dollars in it for the first time. [3]

(a) on 1st day of 2025 : $FV = PV\left(1+\frac{0.048}{12}\right)^{12} = h(1.04907\ldots)$

$r \approx 1.049$ (4 significant figures)

(b) $2h = h\left(1+\frac{0.048}{12}\right)^{12t}$ $t = \#$ of years

$2 = 1.004^{12t} \Rightarrow t \approx 14.469 \Rightarrow 14$ yrs $+ 0.469$ months

$0.469(12) \approx 5.633$ months $\Rightarrow 6$ months

$2h$ dollars first in account July 2038

OR

$PV = 1$

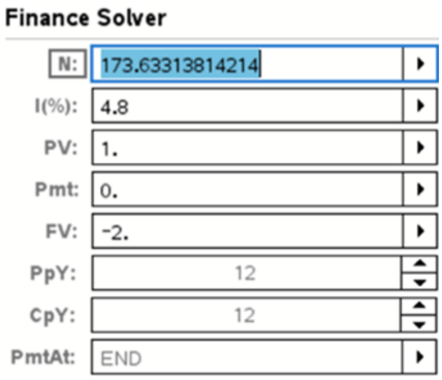
$FV = -2$

$I\% = 4.8$

$P/Y = 4$

$C/Y = 4$

$n = 173.633\ldots$ (payment periods)



$\frac{173.633\ldots}{12} \approx 14.469 \Rightarrow 14$ yrs $+ 6$ months

$2h$ dollars first in account June 2038

2. [Maximum mark: 4]

Prove the following statement:

If m and n are both odd integers, then $m+n$ is an even integer. [4]

Let $m = 2a + 1$ and $n = 2b + 1$
where a and b are integers

$m + n = 2a + 1 + 2b + 1$
$= 2(a + b + 1)$

Since a, b and 1 are integers then
the sum $a + b + 1 = k$ must be an integer

hence, $m + n = 2k$
thus, $m + n$ must be an even integer
since it has a factor of 2 (divisible by 2)

3. [Maximum mark: 6]

Consider the diagram below with the measure of the angle at vertex A and lengths of line segments $[\mathrm{AB}]$, $[\mathrm{BC}]$ and $[\mathrm{BD}]$ indicated. Find the length of line segment $[\mathrm{CD}]$.

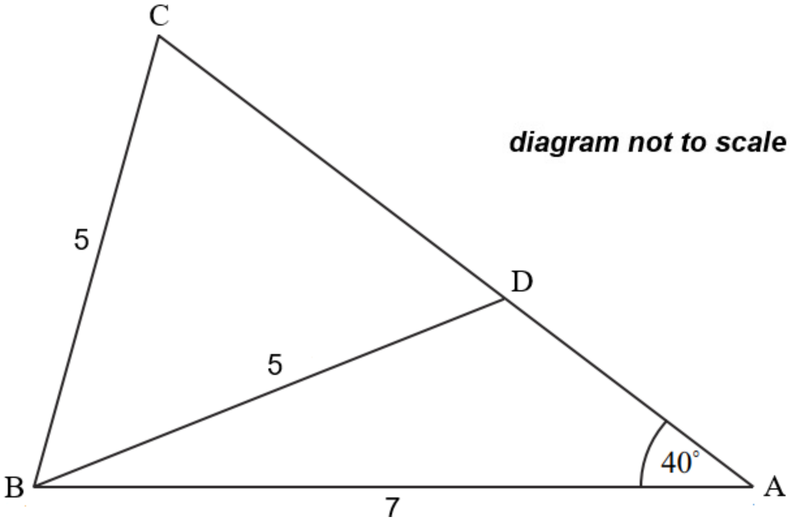
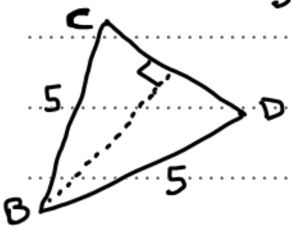


diagram not to scale

Method 1: $\dfrac{\sin C}{7} = \dfrac{\sin 40°}{5}$ sine rule

$\sin C = \dfrac{7\sin 40°}{5} \Rightarrow C = \sin^{-1}\left(\dfrac{7\sin 40°}{5}\right) \approx 64.14...°$



$\cos C = \dfrac{\frac{1}{2}CD}{5} \Rightarrow CD = 2\left[5\cos(64.14...°)\right]$

$CD \approx 4.36$

Method 2: let $AC = x$

$5^2 = 7^2 + x^2 - (2)(7)(x)\cos 40°$ cosine rule

$x^2 - (14\cos 40°)x + 24 = 0 \Rightarrow x^2 - (10.72...)x + 24 = 0$

$x = \dfrac{10.72... \pm \sqrt{(10.72...)^2 - 4(24)}}{2} \Rightarrow x \approx 7.543...$ or $x \approx 3.182...$

let $AD = y$: $5^2 = 7^2 + y^2 - (2)(7)(y)\cos 40°$ [same equation as above]

$y \approx 7.543...$ or $y \approx 3.182...$

hence, $AC \approx 7.543...$ and $AD \approx 3.182...$

$CD = AC - AD = 7.543... - 3.182... \Rightarrow CD \approx 4.36$

4. [Maximum mark: 7]

A particle moves in a straight line. At time t seconds, the particle's displacement from a fixed point O is s meters. The particle's velocity v (in meters per second) is given by $v = \sin\left(\frac{t}{2}\right), t \geq 0$.

When $t = 0$, $s = 2$ meters.

(a) Express the displacement s as a function of time t. [5]

(b) Find the total distance the particle travels from $t = 0$ seconds to $t = 10$ seconds. [2]

(a) $s(t) = \int v(t)\,dt = \int \sin\left(\frac{t}{2}\right)dt = -2\cos\left(\frac{t}{2}\right) + C$

given $s(0) = 2 \Rightarrow -2\cos(0) + C = 2 \Rightarrow C = 4$

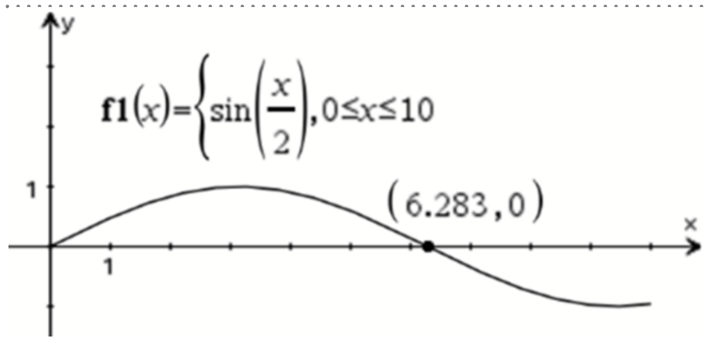
thus, $s(t) = -2\cos\left(\frac{t}{2}\right) + 4$

(b) total distance travelled from $t=0$ to $t=10$ $= \int_0^{10} \left|\sin\left(\frac{t}{2}\right)\right| dt \approx 6.57$ m

OR

total distance $= \int_0^{2\pi} \sin\left(\frac{t}{2}\right) dt - \int_{2\pi}^{10} \sin\left(\frac{t}{2}\right) dt$

$= 4 - (-2.567\ldots) \approx 6.57$ m



5. [Maximum mark: 6]

When $\left(1+\dfrac{2x}{3}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x, the coefficient of x^2 is 68.

(a) Find the value of n. [5]

(b) Hence, find the coefficient of x^3. [1]

(a) $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

coefficient of x^2 for $\left(1+\frac{2x}{3}\right)^n$ is $\binom{n}{2}(1)^{n-2}\left(\frac{2}{3}\right)^2 = 68$

$\dfrac{n!}{2!\,(n-2)!} \cdot 1 \cdot \dfrac{4}{9} = 68 \Rightarrow n(n-1) = 306$

$n^2 - n - 306 = 0 \Rightarrow \cancel{n = -17}$ or $n = 18$

(b) coefficient of $x^3 = \binom{18}{3}\left(\frac{2}{3}\right)^3 = 816\left(\frac{8}{27}\right) = \dfrac{2176}{9}$

$\boxed{\text{or } 241.\overline{7}}$

6. [Maximum mark: 6]

Find the equation of the line that is tangent to the curve $x^3y^2 = \cos(\pi y)$ at the point $(-1, 1)$. Express your answer in the form $y = mx + c$.

$$\frac{d}{dx}\left[x^3y^2\right] = \frac{d}{dx}\left[\cos(\pi y)\right]$$

$$3x^2y^2 + 2x^3y\frac{dy}{dx} = -\pi\sin(\pi y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(2x^3y + \pi\sin(\pi y)\right) = -3x^2y^2 \Rightarrow \frac{dy}{dx} = \frac{-3x^2y^2}{2x^3y + \pi\sin(\pi y)}$$

at $(-1,1)$: $\frac{dy}{dx} = \frac{-3(1)(1)}{2(-1)(1) + \pi\sin\pi} = \frac{-3}{-2+0} = \frac{3}{2}$

equation of line: $y - 1 = \frac{3}{2}(x - (-1))$

$$y = \frac{3}{2}x + \frac{5}{2}$$

7. [Maximum mark: 7]

A continuous random variable X has a probability density function given by $f(x)$ such that

$$f(x)=\begin{cases} k(x+2)^2, & -3\le x<0 \\ k, & 0\le x<2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of k. [2]

(b) Hence, find

(i) the mean of X;

(ii) the median of X. [5]

(a) $k\int_{-3}^{0}(x+2)^2\,dx + \int_{0}^{2}k\,dx = 1$

$3k + 2k = 1 \Rightarrow k = \frac{1}{5}$

(b) (i) $E(X) = \frac{1}{5}\int_{-3}^{0}x(x+2)^2\,dx + \frac{1}{5}\int_{0}^{2}x\,dx$

$= -\frac{9}{20} + \frac{2}{5} \Rightarrow E(X) = -\frac{1}{20}$ $[-0.05]$

(ii) let median $= m$

since $\frac{1}{5}\int_{-3}^{0}(x+2)^2\,dx = \frac{3}{5} > \frac{1}{2}$ then $-3 < m < 0$

$\frac{1}{5}\int_{-3}^{m}(x+2)^2\,dx = \frac{1}{2} \Rightarrow \int_{-3}^{m}(x+2)^2\,dx = \frac{5}{2}$

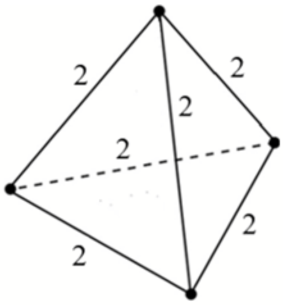
median ≈ -0.134

$$\text{nSolve}\left(\frac{1}{5}\cdot\int_{-3}^{m}(x+2)^2\,dx=\frac{1}{2},m\right)$$

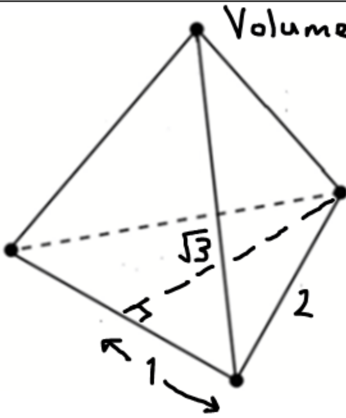
-0.133744421591

8. [Maximum mark: 7]

(a) A regular tetrahedron is a pyramid where all four faces are equilateral triangles. Show that a regular tetrahedron where each edge is 2 cm has a volume of $\frac{2\sqrt{2}}{3}$ cm^3. [3]



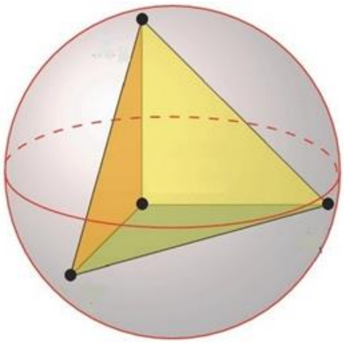
(b) A sphere encloses the tetrahedron from (a) such that all four vertices of the tetrahedron are on the sphere's surface. Find the volume of the sphere. [4]

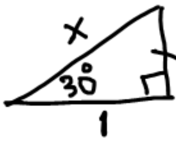


Volume of a right pyramid $= \frac{1}{3}Ah$

A = area of base

$= \sqrt{3}$ cm^2

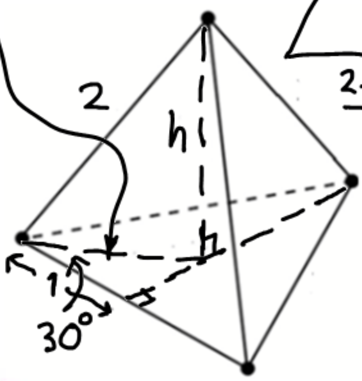


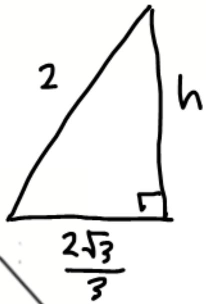


$\cos 30° = \frac{1}{x}$

$\frac{\sqrt{3}}{2} = \frac{1}{x}$

$x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$





$h^2 = 2^2 - \left(\frac{2\sqrt{3}}{3}\right)^2$

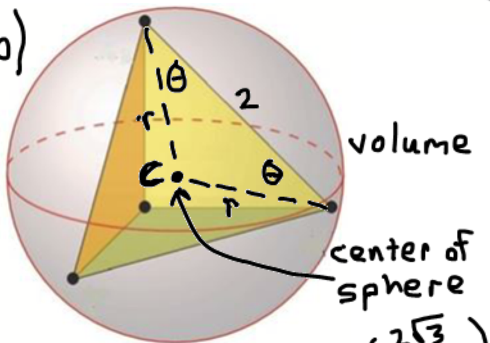
$h^2 = \frac{8}{3} \rightarrow h = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{6}}{3}$ cm

$\text{Volume} = \frac{1}{3}\left(\sqrt{3}\right)\left(\frac{2\sqrt{6}}{3}\right)$

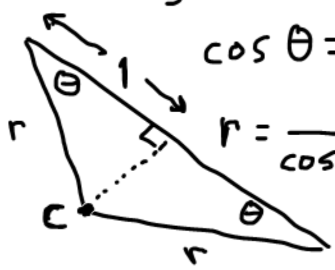
$= \frac{2}{9}\sqrt{18} = \frac{2}{9}\left(3\sqrt{2}\right)$

$\text{Volume} = \frac{2\sqrt{2}}{3}$ cm^3 Q.E.D.

(b)

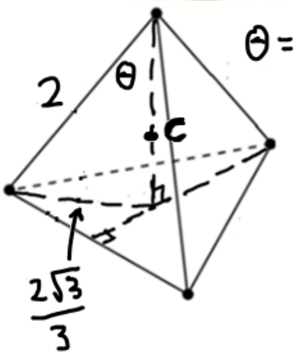


volume of sphere $= \frac{4}{3}\pi r^3$



$\cos\theta = \frac{1}{r}$

$r = \frac{1}{\cos(35.26\ldots°)} \approx 1.2247\ldots$ cm



$\theta = \sin^{-1}\left(\frac{\frac{2\sqrt{3}}{3}}{2}\right)$

$\approx 35.26\ldots°$

$\text{Volume} = \frac{4}{3}\pi\left(1.2247\ldots\right)^3$

$\approx 7.6953\ldots \approx 7.70$ cm^3

9. [Maximum mark: 7]

If $C = \arctan(3) + \arcsin\left(\frac{5}{13}\right)$, find the **exact** value of $\cos C$. [7]

let $A = \arctan(3)$ and $B = \arcsin\left(\frac{5}{13}\right)$

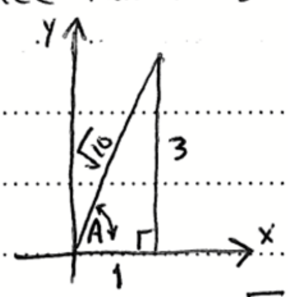
if $C = A + B$, then $\cos C = \cos(A+B)$

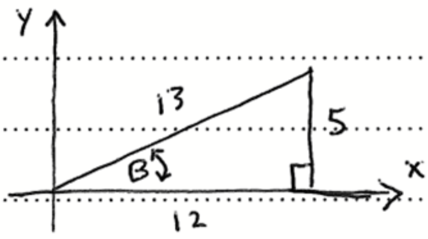
$= \cos A \cos B - \sin A \sin B$

$B = \arcsin\left(\frac{5}{13}\right) \rightarrow \sin B = \frac{5}{13}$

need to find exact values for $\cos A$, $\cos B$ and $\sin A$

since $\tan A = 3 > 0$ and $\sin B = \frac{5}{13} > 0$ both A and B are in first quadrant





$\sin A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$ $\qquad$ $\sin B = \frac{5}{13}$

$\cos A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\qquad$ $\cos B = \frac{12}{13}$

substituting: $\cos C = \left(\frac{\sqrt{10}}{10}\right)\left(\frac{12}{13}\right) - \left(\frac{3\sqrt{10}}{10}\right)\left(\frac{5}{13}\right)$

$\cos C = \frac{-3\sqrt{10}}{130}$

Do **not** write solutions on this page.

Section B (54 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

worked solution on next page →

10. [Maximum mark: 19]

A large farm produces eggs that are packaged in boxes with 12 eggs in each box. The probability that a single egg is cracked is 0.017. A random box is selected and the 12 eggs in it are inspected.

(a) Find the probability that exactly one egg in the box is cracked. [3]

(b) A box fails inspection if at least two eggs in it are cracked. Find the probability that the randomly selected box passes (does not fail) inspection. [3]

The weights of individual eggs are normally distributed with a mean of 58 grams and a standard deviation of 4.7 grams. Before the eggs are packed in boxes a very large number of them are placed in a sorting bin.

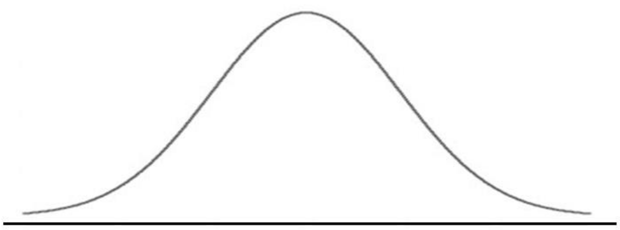
(c) One egg is chosen at random from the bin. Find the probability that this egg

(i) weighs less than 64 grams;

(ii) weighs between 52 grams and 64 grams. [4]

(d) 10% of the eggs in the bin weigh less than w grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating w, and shading the appropriate region.



(ii) Find the value of w. [4]

(e) An egg selected from the sorting bin is accepted to be put into a box if its weight lies between 52 grams and 64 grams. 12 eggs are randomly selected from the sorting bin. Find the probability that all the eggs are accepted to be put in a box. [2]

(f) Eggs from the farm are either white or brown. The colour of an egg has no effect on its weight. The probability an egg is white is 0.28. Find the probability that an egg randomly selected from the sorting bin is accepted to be put in a box and is brown. [3]

[see GDC images for Q.10 solution on next page]

10.

(a) X is random variable representing # of cracked eggs in a box

$$X \sim B(12, 0.017)$$

$$P(x=1) = {}_{12}C_1 (0.017)^1 (1-0.017)^{11}$$

$$P(X=1) \approx \underline{\underline{0.169}}$$

(b) probability a box passes inspection $= P(X \leq 1) = P(x=0) + P(x=1)$

$$P(x=0) = (1-0.017)^{12} = (0.983)^{12} \approx 0.814033\ldots$$

probability box passes $= 0.814033\ldots + 0.168935\ldots \approx 0.982968\ldots$

$$\approx \underline{\underline{0.983}}$$

(c) X is random variable representing weights of eggs

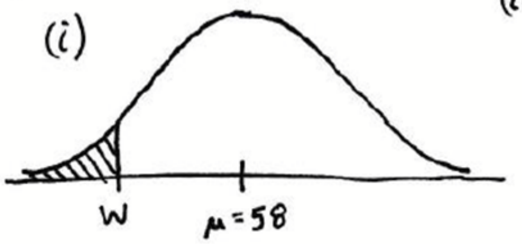
$$X \sim N(58, 4.7^2) \qquad \mu = 58, \ \sigma = 4.7$$

(i) $P(x < 64) \approx \underline{\underline{0.899}}$

(ii) $P(52 < x < 64) \approx \underline{\underline{0.798}}$

(d) $P(x < w) = 0.10$

(i)



(ii) z-value for $P(x < w) = 0.1$ is $z \approx -1.28155$

$$z = \frac{x-\mu}{\sigma} \rightarrow -1.28155 = \frac{x-58}{4.7} \rightarrow x \approx 51.9767$$

$$\underline{\underline{w \approx 52.0}}$$

(e) $P(52 < x < 64) \approx 0.798255\ldots$

probability all 12 eggs are "accepted" (weight between 52 and 64 grams) is equal to $(0.798255\ldots)^{12} \approx \underline{\underline{0.0669}}$

(f) $P(\text{brown}) = 1 - 0.28 = 0.72$

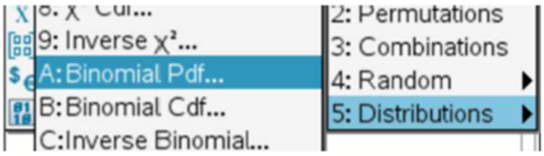
$$P(\text{brown} \cap \text{accepted}) = P(\text{brown}) \cdot P(\text{accepted})$$

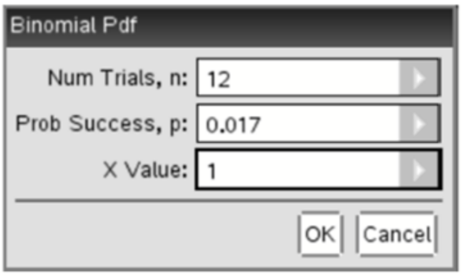
$$= (0.72)(0.798255\ldots)$$

$$\approx \underline{\underline{0.574}}$$

Q.10 GDC images

(a)

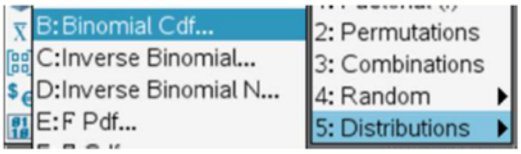


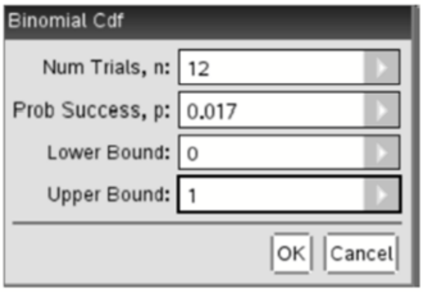


binomPdf(12,0.017,1) 0.168935

nCr(12,1)· 0.017· (1−0.017)^{11} 0.168935

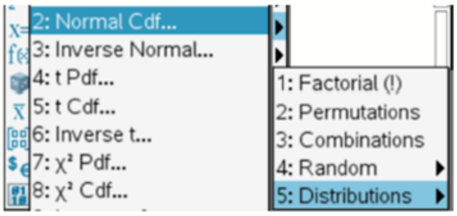
(b)

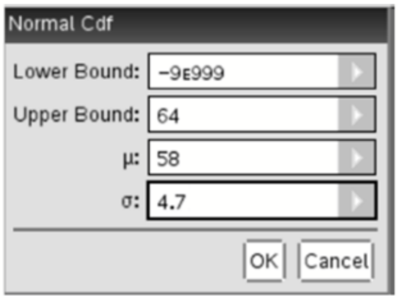




binomCdf(12,0.017,0,1) 0.982968

(c)

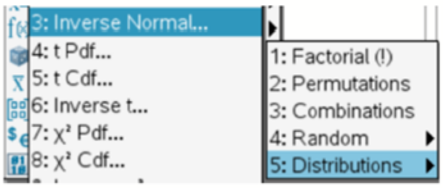




normCdf(−9.E999,64,58,4.7) 0.899127

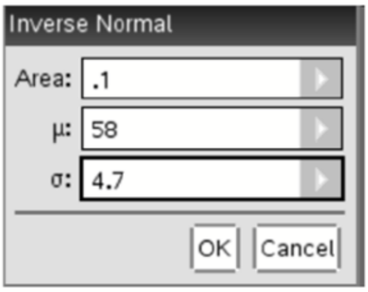
normCdf(52,64,58,4.7) 0.798255

(d) (ii)



invNorm(0.1,0,1) -1.28155

OR



invNorm(0.1,58,4.7) 51.9767

11. [Maximum mark: 20]

(a) The plane Π_1 contains the points $A(2,4,2)$, $B(-3,5,0)$ and $C(-1,5,1)$. Show that $x+y-2z=2$ is a Cartesian equation for plane Π_1. [3]

(b) A second plane Π_2 has Cartesian equation $3x+y+2z=-4$.

(i) Deduce that planes Π_1 and Π_2 intersect at a line.

(ii) Find the angle between planes Π_1 and Π_2. [5]

(c) A third plane Π_3 has Cartesian equation $2x+y+cz=k$ where $c,k\in\mathbb{R}$. Find the conditions on c and k for which

(i) the three planes have no point of intersection (i.e. no points lie on all three planes);

(ii) the three planes intersect at a single point;

(iii) the three planes intersect at a line. [8]

(d) For the case where the three planes intersect at a line, find a Cartesian equation for the line. [4]

- worked solution -

11. (a) $\vec{AB}=\begin{pmatrix}-3\\5\\0\end{pmatrix}-\begin{pmatrix}2\\4\\2\end{pmatrix}=\begin{pmatrix}-5\\1\\-2\end{pmatrix}$ $\quad\vec{AC}=\begin{pmatrix}-1\\5\\1\end{pmatrix}-\begin{pmatrix}2\\4\\2\end{pmatrix}=\begin{pmatrix}-3\\1\\-1\end{pmatrix}$

$$\vec{n}=\begin{pmatrix}-5\\1\\-2\end{pmatrix}\times\begin{pmatrix}-3\\1\\-1\end{pmatrix}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\-5&1&-2\\-3&1&-1\end{vmatrix}=\vec{i}\begin{vmatrix}1&-2\\1&-1\end{vmatrix}-\vec{j}\begin{vmatrix}-5&-2\\-3&-1\end{vmatrix}+\vec{k}\begin{vmatrix}-5&1\\-3&1\end{vmatrix}$$

$$=\vec{i}+\vec{j}-2\vec{k}=\begin{pmatrix}1\\1\\-2\end{pmatrix}$$

plane Π_1: $x+y-2z=d$ substitute a pt., e.g. A, to find d

$2+4-2(2)=d \rightarrow d=2$

thus, equation for Π_1 is $x+y-2z=2$ Q.E.D.

(b)(i) Π_1 and Π_2 are not parallel since their normal vectors, $\begin{pmatrix}1\\1\\-2\end{pmatrix}$ and $\begin{pmatrix}3\\1\\2\end{pmatrix}$, are not parallel. Therefore, Π_1 and Π_2 must intersect at a line.

[solution for Q.11 continued on next page]

[solution for Q.11 continued]

(ii) the angle θ between the planes Π_1 and Π_2 will be equal to the angle between the normal vectors of the planes.

$$\theta = \cos^{-1}\left(\frac{\begin{pmatrix}1\\1\\-2\end{pmatrix}\cdot\begin{pmatrix}3\\1\\2\end{pmatrix}}{\sqrt{1+1+4}\sqrt{9+1+4}}\right) = \cos^{-1}\left(\frac{0}{\sqrt{6}\sqrt{14}}\right) \rightarrow \theta = 90^\circ$$

(c) using row operations with augmented matrix

$$\left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 3 & 1 & 2 & -4\\ 2 & 1 & c & k\end{array}\right] \quad \xrightarrow{R3-2R1\to R3} \quad \left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 3 & 1 & 2 & -4\\ 0 & -1 & c+4 & k-4\end{array}\right] \quad \xrightarrow{R2-3R1\to R2} \quad \left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 0 & -2 & 8 & -10\\ 0 & -1 & c+4 & k-4\end{array}\right]$$

$$\xrightarrow{R2/-2\to R2} \quad \left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 0 & 1 & -4 & 5\\ 0 & -1 & c+4 & k-4\end{array}\right] \quad \xrightarrow{R3+R2\to R3} \quad \left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 0 & 1 & -4 & 5\\ 0 & 0 & c & k+1\end{array}\right]$$

(i) no point of intersection (no solution) when $c = 0$ and $k \neq -1$

(ii) planes intersect at a single point when $c \neq 0$ and $k \in \mathbb{R}$

(iii) planes intersect at a line when $c = 0$ and $k = -1$

(d) $\left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 0 & 1 & -4 & 5\\ 0 & 0 & c & k+1\end{array}\right]$ planes intersect at a line when $c = 0$ and $k = -1$ (infinite solutions) $\left[\begin{array}{ccc|c}1 & 1 & -2 & 2\\ 0 & 1 & -4 & 5\\ 0 & 0 & 0 & 0\end{array}\right]$

let $z = \lambda$ then $0x + y - 4\lambda = 5 \rightarrow y = 5 + 4\lambda$

and $x + (5+4\lambda) - 2\lambda = 2 \rightarrow x = -3 - 2\lambda$

parametric eqns. for line of intersection $\begin{cases} x = -3-2\lambda \\ y = 5+4\lambda \\ z = \lambda \end{cases}$

solve for λ in each equation

$$\lambda = \frac{x+3}{-2}, \quad \lambda = \frac{y-5}{4}, \quad \lambda = z$$

thus, a Cartesian equation of line of intersection is $\dfrac{x+3}{-2} = \dfrac{y-5}{4} = z$

[note: this equation for line is not unique; there are other correct answers]

12. [Maximum mark: 15]

(a) Using the appropriate double angle identity for cosine, prove that $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$. [2]

(b) Hence, find $\int \cos^2 x \, dx$. [3]

Consider functions $f(x) = \cos^2 x$ and $g(x) = \sec^2 x - \frac{3}{2}$ each with domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(c) Find the **exact** values of the x-coordinates of the points where the graphs of f and g intersect. [5]

(d) Let R be the region enclosed by the graphs of f and g. Find the **exact** area of R. [5]

- worked solution -

(a) $\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos 2\theta \rightarrow \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$ Q.E.D.

(b) $\int \cos^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x \, dx$

$= \frac{1}{2}x + \frac{1}{2}\left(\frac{1}{2}\sin 2x\right) = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

(c) $\cos^2 x = \sec^2 x - \frac{3}{2}$

$\cos^2 x = \frac{1}{\cos^2 x} - \frac{3}{2}$

$\cos^2 x = \frac{2 - 3\cos^2 x}{2\cos^2 x}$

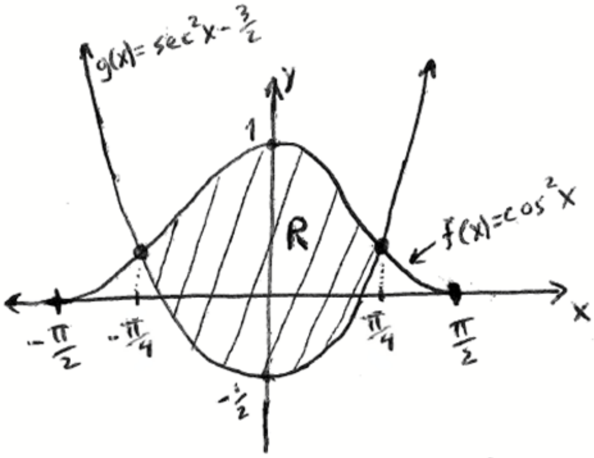
$2\cos^4 x + 3\cos^2 x - 2 = 0$

$(2\cos^2 x - 1)(\cos^2 x + 2) = 0$

$\cos^2 x = \frac{1}{2} \qquad \cos^2 x \neq -2$

$\cos x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$x = \pm\frac{\pi}{4}$



x-coordinates of points of intersection are $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$

(d) area of $R = 2\int_0^{\pi/4} \left(\cos^2 x - \left(\sec^2 x - \frac{3}{2}\right)\right) dx$

$= 2\left[\int_0^{\pi/4} \cos^2 x \, dx - \int_0^{\pi/4} \left(\sec^2 x - \frac{3}{2}\right) dx\right]$

$= 2\left\{\left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi/4} - \left[\tan x - \frac{3}{2}x\right]_0^{\pi/4}\right\}$

$= 2\left\{\left[\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0)\right] - \left[\left(1 - \frac{3\pi}{8}\right) - (0)\right]\right\}$

$= 2\left(\frac{4\pi}{8} - \frac{3}{4}\right) = \frac{2\pi - 3}{2}$ units^2 Q.E.D.